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Quantitative Methods for

Project Management

ProjectManagementIQ™

International Institute for Learning, Inc.

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About the Author

Dr. Frank T. Anbari (Ph.D. Project Management and Quality Enhancement, MBA, MS Engineering, P.E., and PMP) developed and taught courses and provided consulting services in project management, quality management, information technology and telecommunications for several universities, private industry and public sector organizations.

Dr. Anbari gained extensive industrial experience serving as Manager - Information Technology, Project Manager - Technical Systems, Senior Project Engineer, Manager of Applications Development, Maintenance and Support, Manager of Analysis and Reporting, and Continuous Quality Improvement Facilitator, at the National Railroad Passenger Corporation (Amtrak), as Lead Civil Engineer at Day and Zimmermann, Inc. and as Project Engineer at American Water Works Service Company, Inc.

Dr. Anbari teaches in the Graduate and the Project Management Certificate Programs at Penn State University and in the MBA Program at Drexel University. He served as examiner for the Malcolm Baldrige National Quality Award, 1993-1995. He serves as executive director for the International Institute for Learning, Inc.

Preface

This booklet provides the reader with a summary of selected quantitative methods procedures. These procedures are selected in view of their wide applicability in project management. The aim of this booklet is to facilitate the understanding of the mechanics of these procedures and their underlying theory.

As a result, it is anticipated that the reader will be able to devote more time and effort to understanding the theories behind these procedures and expanding their applications to their own projects. There is no substitute for profound knowledge of quantitative methods tools. Such knowledge is imperative for excellence in project management.

Frank T. Anbari
1997

Quantitative Methods for Project Management

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Populations, Processes and Samples

The set of all **observations** that characterize some phenomenon is referred to as the **population**, the **universe** or the **process**.

The set of observations selected from the population (or universe, or process) is referred to as the **sample**.

By studying the sample an **inference** may be made about the population (or universe, or process).

The Need for Sampling

To study and analyze a process, sampling is needed. Conducing a **census** of the population is not normally appropriate because of the following factors:

- Economic factors
- Time factor
- Population size
- Partially inaccessible populations/processes
- Destructive nature of the observation
- Accuracy

Parameters and Statistics

Summary measures for the population, the universe or the process are called **parameters**. The are usually represented by lower case Greek letters. Following are some examples of population or process parameters:

- Population or process mean, denoted by μ
- Population or process variance, denoted by σ^2
- Population or process standard deviation, denoted by σ

The number of observations in the population, the universe or the process is called the population size and denoted by N.

Summary measures for the sample are called **statistics**. The are usually represented by lower case English letters. Following are some examples of sample statistics:

- Sample mean, denoted by $\bar{X}$
- Sample variance, denoted by s^2
- Sample standard deviation, denoted by s

The number of observations in the sample is called the sample size and denoted by n.

Descriptive Statistics

Information on the **population**, the **universe,** the **process** or the **sample** can be summarized using measures of:

- Central tendency (location)
  - Mean
  - Median
  - Mode

- Variability (dispersion)
  - Range
  - Variance
  - Standard Deviation

- Shape
  - Pearson's coefficient of skewness
  - Down side variance

Measures of central tendency (location)

These measures provide values around which the data tends to congregate. Widely used measures of central tendency or location include:

- Mean
- Median
- Mode

Mean

The mean is the sum of measurements divided by the number of measurements:

The population or process mean is denoted by μ:

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

The sample mean is denoted by $\bar{x}$ (x-bar):

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Median

The median is the value having as many observations above it as there are below it, when the observations are arranged in an increasing or decreasing sequence of values. The median can be denoted by M or Md.

If an even number of observations is collected from the process, then the median is the mean of the two observations in the middle, when the observations are arranged in an increasing or decreasing sequence of values.

Mode

The mode is the most frequently occurring observation the data set under consideration. The mode can be denoted by Mo.

If the observations have two modes, then the data is said to have a bi-modal distribution.

If the observations have three, or more, modes, then the mode is no longer a viable measure of central tendency.

Comments on measures of central tendency

The mean, median and mode have the same dimension, or unit of measure, as the original data. The mean is the most widely used measure of central tendency. However, it tends to be influenced by extreme observations. In such cases, the median may be a more appropriate measure of central tendency. The median is a more "democratic" measure. It give each observation one "vote" regardless of its magnitude. The median considers only whether the observation is larger or smaller than the value in the middle, regardless of the magnitude of the observation. Thus, the median does not use all the information available about the observations.

As examples, the median is usually used in reporting summaries of salaries, home prices, and ages.

Example

In a particular project, a sample of ten change orders have the following prices, in thousands of dollars: 2, 2, 2, 3, 3, 4, 5, 7, 10, 12.

The mean is:

$$\bar{x} = \frac{50}{10} = 5$$

The median is: 3.5

The mode is: 2.

Measures of variability (dispersion)

These measures provide indications of how the data is dispersed. Widely used measures of variability, variation, or dispersion include:

- Range
- Variance
- Standard Deviation

Range

The range is the difference between the largest measurement and the smallest measurement. The range is denoted by R or r.

$$R = x_{max} - x_{min}$$

The range does not use all available observations. It uses only two extreme values. It has the same dimension, or unit of measure, as the original data.

Variance

The population or process variance (σ^2) of N measurements is the sum of the squared deviations from the mean divided by N. The population or process variance is denoted by σ^2:

$$\sigma^2 = \frac{\sum_{i=1}^{N}\left(x_i - \mu\right)^2}{N}$$

The sample variance (s^2) of n measurements is the sum of the squared deviations from the mean divided by (n-l). The sample variance is denoted by s^2:

$$s^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}$$

The divisor n-1 is used in the sample variance (s^2) to obtain an unbiased estimate of the population variance (σ^2).

The dimension, or unit of measure, of the variance is the square of the dimension of the original data.

Standard Deviation

The standard deviation is the positive square root of the variance.

The population or process standard deviation is denoted by σ:

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}}$$

The sample standard deviation is denoted by s:

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}}$$

The standard deviation has the same dimension, or unit of measure, as the original data. The standard deviation can be approximated to an order of magnitude by the range over six. Some prefer to divide the range over four.

Example

In a the project discussed earlier, a sample of ten change orders have the following prices, in thousands of dollars: 2, 2, 2, 3, 3, 4, 5, 7, 10, 12.

The range is:

$$R = 12 - 2 = 10$$

The sample variance is:

$$s^2 = \frac{114}{10-1} = 12.67$$

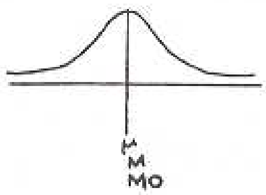
The sample standard deviation is:

$$s = \sqrt{12.67} = 3.56$$

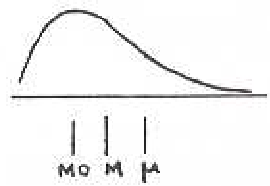
Measures of shape (skewness)

Skewness reflects a positional comparison of the measures of central tendency. Measures of skewness provide indications of whether the data is symmetrical or skewed, and how much the data is skewed. These measures include:

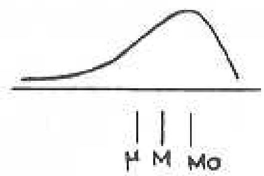
- Pearson's coefficient of skewness
- Down side variance



Symmetrical



Skewed to the right
(Positively skewed)



Skewed to the left
(Negatively skewed)

Pearson's coefficient of skewness (γ):

$$\gamma = \frac{3(\mu - M)}{\sigma}$$

If: $\gamma > 0$ the distribution is positively skewed (skewed to the right)

If: $\gamma < 0$ the distribution is negatively skewed (skewed to the left)

If: $\gamma = 0$ the distribution is symmetrical

Down-Side Variance (DSV)

The down-side variance (DSV) is calculated using only the values smaller than the mean:

$$DSV = \frac{\sum_{i=1}^{n}\left(x_{is} - \bar{x}\right)^2}{n-1}$$

where x_{is} represents the values smaller than the mean, and n represents the number of measurements in the entire data set.

The following ratio can be calculated to represent skewness (SK):

$$SK = \frac{s^2}{2(DSV)}$$

If:	$SK > 1$	the distribution is skewed to the right (positively skewed)
If:	$SK < 1$	the distribution is skewed to the left (negatively skewed)
If:	$SK = 1$	the distribution is symmetrical

Example

In a the project discussed earlier, a sample of ten change orders have the following summary measures, in thousands of dollars: mean = 5, median = 3.5, and standard deviation = 3.56.

Pearson's coefficient of skewness is:

$$\gamma = \frac{3(5-3.5)}{3.56} = 1.26$$

The down-side variance (DSV) is 4, and skewness (SK) is:

$$\text{SK} = \frac{12.67}{2\text{x}4} = 1.58$$

Probability

Definition

The probability of an event "A" is value that ranges between "0" and "1". It is denoted as P(A), and defined as:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$$

From the above definition, the following statements can be made:

$$P\text{ (Impossible event)} = 0$$

$$P\text{ (Certain event)} = 1$$

$$0 \leq P\text{ (Any Event)} \leq 1$$

The sum of the probabilities of "n" mutually exclusive and collectively exhaustive events A_i equals to "1":

$$\sum_{i=1}^{n} A_i = 1$$

Complementary events

$$P(\bar{A}) = 1 - P(A)$$

The multiplication rule for independent events

$$P(A \cap B) = P(A) \,.\, P(B)$$

$$P(A \cap B \cap C \ldots) = P(A) \,.\, P(B) \,.\, P(C) \ldots$$

Dependent events

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

The addition rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

The addition rule for mutually exclusive events

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B \cup C \ldots) = P(A) + P(B) + P(C) \ldots$$

Applications in risk management

In a given company, historical evidence indicates that a particular risk event, such as late deliveries of material or equipment to the project site, has a 0.05 probability. Assuming this risk event to be independent of other events in the project, the probability of its occurrence remains constant, until such material or equipment is delivered.

Applications in quality management

Series Reliability

Consider a project in which two components are to operate in a series. The reliability of one component is 0.90, and the reliability of the other component is 0.95. The reliability of the assembly of these two components in a series is:

$P(A \cap B) = P(A) \,.\, P(B) = 0.90 \times 0.95 = 0.885$

Thus, series reliability is less than the reliability of any component in the series.

Parallel Reliability (Redundancy)

Consider the same project in which two components are to operate in parallel. As long as one component is operational, the other component is redundant. The reliability of one component is 0.90, and the reliability of the other component is 0.95. The reliability of the assembly of these two components in parallel is calculated as follows:

$P(\bar{A}) = 1 - P(A) = 1 - 0.90 = 0.10$

$P(\bar{B}) = 1 - P(B) = 1 - 0.95 = 0.05$

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \,.\, P(\bar{B}) = \; = 0.1 \times 0.05 = 0.005$

Parallel Reliability = 1 - 0.005 = 0.995

Thus, parallel reliability is higher than the reliability of any component in the parallel assembly.

Probability Distributions

Binomial Distribution

- n identical independent trials
- Two outcomes
- Constant probability of success, denoted by p
- Constant probability of failure, denoted by q = 1 - p
- Probability of x successes in n trials:

$$P(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

Applications in quality management

In constructing quality control charts for attributes, the following are based on the binomial distribution:

- np-chart: Number of defectives when subgroup size is constant
- p-chart: Fraction defective chart when subgroup size is constant or variable

Poisson Distribution

- Number of occurrences of an event during an interval is independent among intervals
- Probability of occurrence is constant among intervals
- Probability of occurrence in a small interval is small and proportional to the length of the interval
- Probability of x occurrence in a given interval:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where: λ = Mean number of occurrences in a given interval

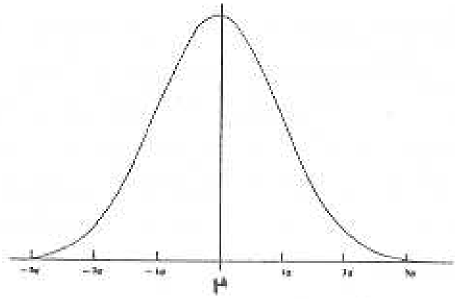
e = 2.71828...

Applications in quality management

In constructing quality control charts for attributes, the following are based on the Poisson distribution:

- c-chart: Total number of defects per unit when subgroup size is constant
- u-chart: Average number of defects per unit when subgroup size is variable

Normal Distribution



- Symmetrical
- Uni-modal
- Extends from $-\infty$ to $+\infty$
- Defined by two parameters: μ and σ
- Total area under the curve always equals 1

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left[(x-\mu)/\sigma\right]^2}{2}}$$

where: $\pi \approx 3.1416$
$e \approx 2.71828$

- Standard normal distribution for $\mu = 0$ and $\sigma = 1$

$$z = \frac{x - \mu}{\sigma}$$

- Selected values for the standard normal distribution are as follows:

Standard deviations from the mean (z)	Area within the (z) Standard deviations	Area in the upper and lower tails
1	0.682 7	0.317 3
2	0.954 5	0.045 5
3	0.997 3	0.002 7
4	0.999 937	0.000 063
5	0.999 999 43	0.000 000 57
6	0.999 999 998	0.000 000 002

Applications in time management:

Program Evaluation and Review Technique (PERT)

Using the Program Evaluation and Review Technique (PERT), a particular project has an expected completion time (length of the critical path) of 20 weeks and a standard deviation of 2 weeks. Based on the assumption that the critical path is normally distributed, the following probabilities can be determined:

Probability of completing the project in less than 20 weeks = 50%
Probability of completing the project in more than 20 weeks = 50%
Probability of completing the project in exactly 20 weeks = 0%

Probability of completing the project in 18 weeks to 22 weeks ≈ 68%
Probability of completing the project in 16 weeks to 24 weeks ≈ 95%
Probability of completing the project in 14 weeks to 26 weeks ≈ 99.7%

Probability of completing the project in less than 22 weeks ≈ 84%
Probability of completing the project in more than 22 weeks ≈ 16%

Probability of completing the project in less than 24 weeks ≈ 97.5%
Probability of completing the project in more than 24 weeks ≈ 2.5%

Probability of completing the project in less than 26 weeks ≈ 99.85%
Probability of completing the project in more than 26 weeks ≈ 0.15%

Applications in quality management

In constructing quality control charts, the center line (CL) is set at the mean, the upper control limit (UCL) is set 3 standard deviations above the center line, and the lower control limit (LCL) is set 3 standard deviations below the center line. Based on the assumption that the points plotted on the control chart are normally distributed, and for a stable process (process in statistical control), the following can be determined:

Percentage of points within the control limits ≈ 99.7%
Percentage of points outside the control limits ≈ 0.3%

Uniform Distribution

- Possible outcomes are equally likely
- Probability is the same in any small sub-interval of a specific interval and zero everywhere else

Triangular Distribution

- Three possible outcomes. These outcomes can be called: a, m and b
- Possible outcomes are equally likely
- Mean of the triangular distribution:

$$\text{Mean} = \frac{a + m + b}{3}$$

Beta Distribution

- Continuous distribution
- Constrained to a finite interval of possible values
- Can be symmetrical, skewed to the right or skewed to the left
- Its mean can be approximated by:

$$\text{Mean} = \frac{a + 4m + b}{6}$$

where:

a = the lower extreme value, such as the optimistic completion time of an activity

m = the most likely value, such as the most likely completion time of an activity

b = the upper extreme value, such as the pessimistic completion time of an activity

- Its variance can be approximated by:

$$\text{Variance} = \left(\frac{b-a}{6}\right)^2$$

- Its standard deviation can be approximated by:

$$\text{Standard deviation} = \frac{b-a}{6}$$

Applications in time management:
Program Evaluation and Review Technique (PERT)

Using PERT, the duration of an activity in a particular project has the following time estimates: optimistic: 3 weeks, most likely: 5 weeks, pessimistic: 7 weeks. The time estimate that should be used for that activity is:

$$\text{Mean} = \frac{3 + 4x5 + 7}{6} = 5 \text{ weeks}$$

Note that when the optimistic and pessimistic estimates are symmetrical around the most likely estimate, then the time estimate for the activity is the same as the most likely estimate.

The standard deviation of the above activity is:

$$\text{Standard deviation} = \frac{7 - 3}{6} = 0.67 \text{ weeks}$$

Using the same approach for an activity that has a time estimate of 6 weeks, an optimistic estimate of 3 weeks, and a pessimistic estimate of 12 weeks. The most likely time estimate for that activity is calculated as follows:

$$6 = \frac{3 + 4m + 12}{6}$$

$$4m = 36 - 15$$

$$m = 3.5 \text{ weeks}$$

Comments on PERT and CPM

In PERT, three estimates of the duration of an activity are obtained: optimistic, most likely, and pessimistic. Their weighted average is calculated, based on the beta distribution approximation shown above. Probabilities of project completion can then be calculated, with the assumption that the duration of the critical path follows the normal distribution. PERT is rarely used in current practice.

In CPM, a single estimate of the duration of an activity is obtained, and considered to be the most likely estimate. No calculations of probabilities of project completion are made.

Sampling and Sampling Distribution

Sampling from a normal population

When a random sample is selected from a normal population, the sample means $\bar{x}$ will also be a normally distributed.

The Central Limit Theorem: Sampling from a non-normal population

When a random sample is selected from a non-normal population, the sample means $\bar{x}$ tend toward a normal distribution, as the sample size increases.

Mean and standard deviation of sampling distribution

- The expected value of the distribution of sample means $E(\bar{x})$ is the population mean μ:

  $$E(\bar{x}) = \mu$$

- The expected variance of the distribution of sample means $V(\bar{x})$ is:

  $$V(\bar{x}) = \frac{\sigma^2}{n}$$

- The expected standard deviation of the distribution of sample means $\sigma_{\bar{x}}$, also refereed to as the standard error, is:

  $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Applications in quality management

In constructing standard quality control charts, the assumption that the points plotted on the control chart are normally distributed is usually made. If the characteristic under consideration is not normally distributed, samples of n items, say 4 or 5 items, are taken. The mean of each sample $\bar{x}$ is calculated and plotted on the control chart. The sample means tend toward a normal distribution, as the sample size increases.

It is important to note that as the sample size increases, the standard deviation of the distribution of sample means decreases. Therefore, the distance between the control limits decrease.

Sum of variances

If the variables under consideration are independent, then the variance of their sum σ_T^2 equals the sum of their variances:

$$\sigma_T^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots$$

Therefore, the standard deviation of their sum σ_T equals the square root of the sum of their variances:

$$\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots}$$

Application in the Program Evaluation and Review Technique (PERT)

The standard deviation of the critical path in PERT is calculated as the square root of the sum of the variances of activities on the critical path:

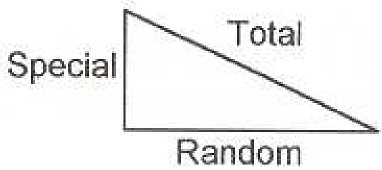
$$\sigma_{CP} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots}$$

Applications in quality management

If sources of variation are independent, the total standard deviation is calculated as the square root of the sum of the variances of the causes contributing to variation.

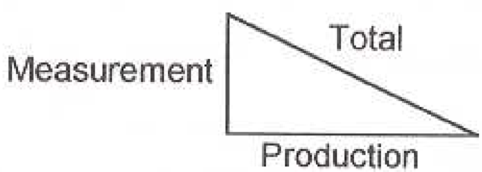
The total standard deviation resulting from random cause variation and special cause variation is:

$$\sigma_{Total} = \sqrt{\sigma^2_{Random} + \sigma^2_{Special}}$$



The total standard deviation resulting from production variation and measurement variation is:

$$\sigma_{Total} = \sqrt{\sigma^2_{Production} + \sigma^2_{Measurement}}$$



Decision Theory and the Expected Value

Decision Theory

Decision makers select among alternative courses of action under conditions that can be classified as certainty, risk, or uncertainty.

Certainty: The Decision-maker knows with certainty the outcome of each alternative.

Risk: The Decision-maker knows all the outcomes of each alternative and the probability of each outcome. The sum of these probabilities is 1.0

Uncertainty: The Decision-maker does not know all the outcomes of each alternative, or does not know the probability of some or all outcome.

In project management, it is more likely to encounter decision-making situations under risk than the other two extremes.

Two types of risk can be defined as follows:

Business risk: This profit or loss risk inherent in any business undertaking.

Pure risk: This risk of only a loss is the result of such events as fire or accidents. It is generally insurable.

Risk decreases as the project progresses, whereas the amount at stake increases. Sensitivity analysis needs to be conducted to determine how the decision may change as a result of changes in the factors affecting it.

Expected Value

The expected value E(x), or the expected monetary value (EMV), is calculated by multiplying each of the "n" outcomes by its probability, then adding the results. It can be formulated as follows:

$$E(x) = \sum_{i=1}^{n} x_i P(x_i)$$

The expected value applies to decision making under risk. Therefore, the sum of probabilities must add up to 1.0. The term expected monetary value (EMV) does not imply that the outcomes are monetary. It can be applied to non-monetary outcomes.

The expected value approach is considered appropriate for rational decision makers when the probabilities of all outcomes can be assessed. When these probabilities cannot be assessed, the following criteria may be used:
Maximax (Optimist) criteria: Find the maximum payoff within every alternative, compare these values and select the alternative with the maximum value.
Maximin (Pessimist) criteria: Find the minimum payoff within every alternative, compare these values and select the alternative with the maximum value.

Application in risk management

Consider a project with four possible profit outcomes: a profit of $200,000, a profit of $100,000, break-even, or a loss of $100,000. Based on historical data for similar projects, probabilities of these outcomes are: 0.5, 0.2, 0.2, and 0.1, respectively. The expected profit for this project is calculated as follows:

Outcome: x_i	Probability: $P(x_i)$	$x.P(x_i)$
$200,000	0.5	$100,000
$100,000	0.2	$ 20,000
$ 0	0.2	0
($100,000)	0.1	($ 10,000)
	-----	--------------
	1.0	$110,000

Applications in time management

The Program Evaluation and Review Technique (PERT) formula for the expected time, or duration, of a task can be thought of as an application of the expected value with the following outcomes and probabilities:

Outcome: x_i		Probability: $P(x_i)$	$x.P(x_i)$
Pessimistic	(a)	1/6	Pessimistic / 6
Most likely	(m)	4/6	4 x Most likely / 6
Optimistic	(b)	1/6	Optimistic / 6

$$\text{Activity time estimate} = \frac{\text{Pessimistic} + 4 \times \text{Most likely} + \text{Optimistic}}{6}$$

PERT and schedule simulation take into consideration the risk aspects of project scheduling.

Probability Trees and Decision Trees

Probability trees and decision tree enhance visualization of various events and their successive outcomes. Probability trees contain events, outcomes, and probabilities. They are used to facilitate calculations of probabilities of various combinations of outcomes. Decision trees contain decisions, events, outcomes, probabilities, and value or cost of various outcomes. They are used to facilitate selection among alternative decisions.

Probability Trees

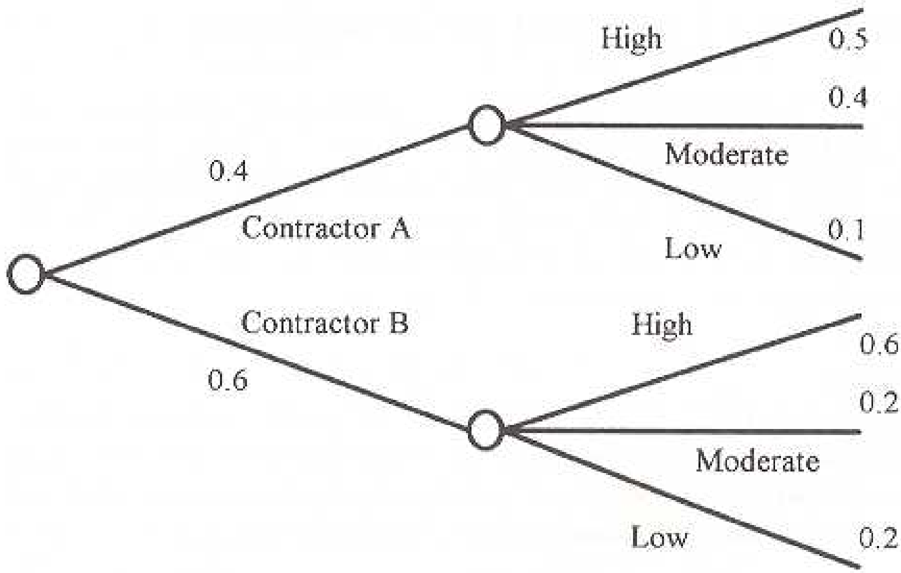
Probability trees are used to facilitate calculations of probabilities of various combinations of outcomes. In a probability tree each event is represented by a circle or a point, each outcome is represented by a branch, and the probability of each outcome is shown alongside its respective branch.

Application in risk management

Two bidders are under consideration for the award of a contract for designing and implementing a new technology project. The probability of selecting contractor A is 0.40, and the probability of selecting contractor B is 0.60. Based on historical data for similar projects, probabilities of customer satisfaction with the results of the project are:

	Contractor A	Contractor B
High	0.5	0.6
Moderate	0.4	0.2
Low	0.1	0.2
	----	-----
Total	1.0	1.0

The probability tree for this project and the probabilities of various combinations of outcomes are as follows:



The probability of selecting contractor A and achieving high customer satisfaction is: 0.4 x 0.5 = 0.20
The probability of selecting contractor A and achieving moderate customer satisfaction is: 0.4 x 0.4 = 0.16
The probability of selecting contractor A and achieving low customer satisfaction is: 0.4 x 0.1 = 0.04

The probability of selecting contractor B and achieving high customer satisfaction is: 0.6 x 0.6 = 0.36
The probability of selecting contractor B and achieving moderate customer satisfaction is: 0.6 x 0.2 = 0.12
The probability of selecting contractor B and achieving low customer satisfaction is: 0.6 x 0.2 = 0.12

The sum of probabilities for all combinations of outcomes is 1.0.

Above calculations are for combinations of outcomes. Probabilities of specific outcomes are as originally stated in the problem. For example, the probability of selecting contractor A is 0.4, and the probability of achieving high customer satisfaction if contractor B is selected is 0.6.

Decision Trees

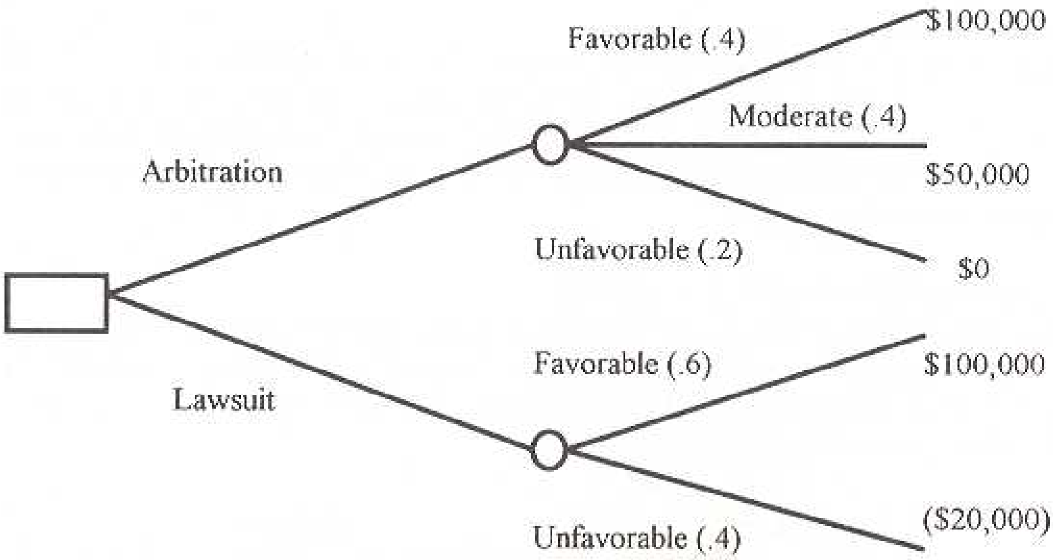
Decision trees are used to facilitate selection among alternative decisions. In a decision tree, each decision is represented by a square, each event is represented by a circle or a point, each outcome is represented by a branch. The probability of each outcome, and its value or cost are shown alongside its respective branch.

Application in risk management

Negotiations failed to resolve a contract dispute resulting from a construction project. One party is considering submitting the matter to binding arbitration or filing a lawsuit for liquidated damages. The outcomes of each action, their probabilities and profits (or losses) are:

Arbitration			Lawsuit		
Outcome	Prob.	Profit	Outcome	Prob.	Profit
Favorable	0.4	$100,000	Favorable	0.6	$100,000
Moderate	0.4	$ 50,000	Unfavorable	0.4	($ 20,000)
Unfavorable	0.2	$ 0			

The decision tree for this contract dispute decision is as follows:



The expected value of each decision is:

Arbitration

Outcome	Prob.	Profit	Prob. x Profit
Favorable	0.4	$100,000	$40,000
Moderate	0.4	$ 50,000	$20,000
Unfavorable	0.2	$ 0	$ 0

Total			$60,000

Lawsuit

Outcome	Prob.	Profit	Prob. x Profit
Favorable	0.6	$100,000	$60,000
Unfavorable	0.4	($ 20,000)	($ 8,000)

Total			$52,000

The expected monetary value of submitting the matter to binding arbitration is $60,000, whereas the expected monetary value of filing a lawsuit for liquidated damages is $52,000. Therefore, it is better for the decision maker to submit the matter to binding arbitration.

Quality Management Tools

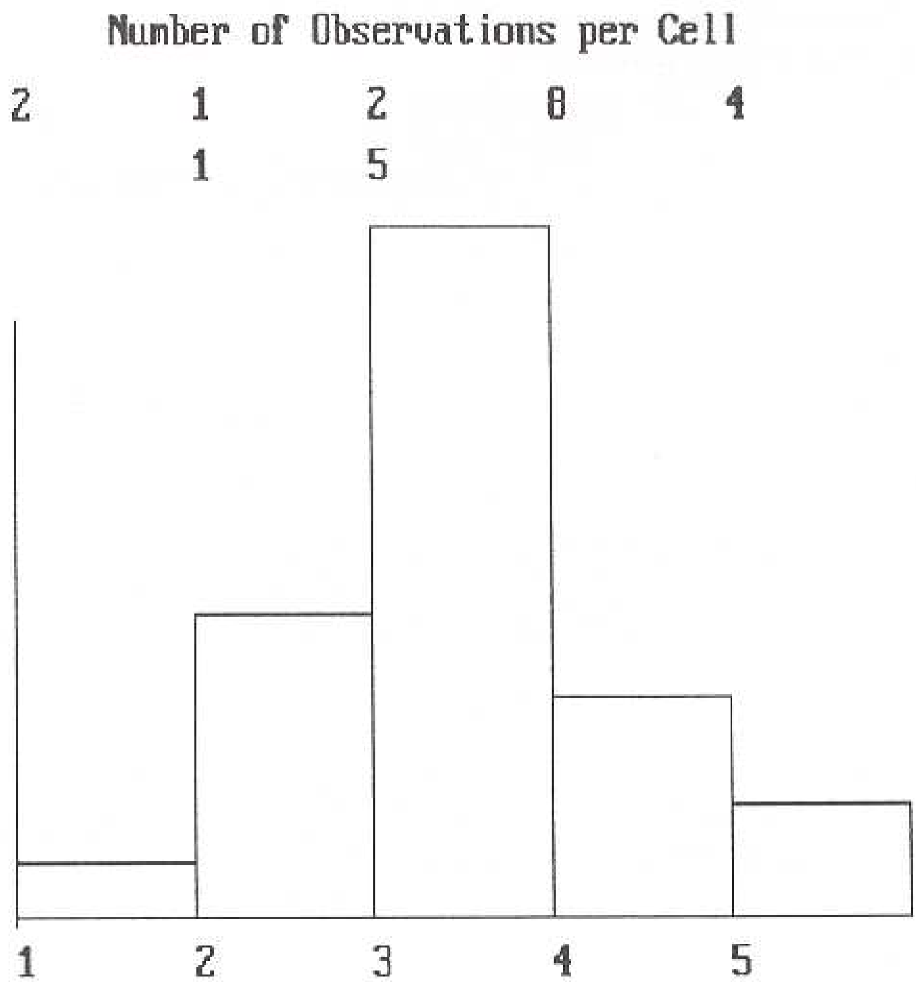
Several tools have been used to plan, enhance and control quality. The following selected tools are widely used for these purposes:

- Histogram
- Pareto analysis
- Cause and effect diagram
- Scatter diagram
- Regression and correlation analysis
- Control chart
- Process capability
- Acceptance sampling

Histogram

The histogram is a graphical representation of the frequency distribution of the variable under consideration. The histogram is usually used to represent variables data. Values of the variable are usually grouped into intervals and shown on the horizontal axis. The frequencies of occurrence of values in various intervals are shown on the vertical axis.

An example of a histogram of the frequencies of response time, in seconds, of a particular computer system follows:



Pareto analysis

The Pareto analysis allows the differentiation between the vital few and the useful many causes of quality problems. It is also referred to as the 80:20 rule or the 20:80 rule. It was developed by Dr. J. M. Juran.

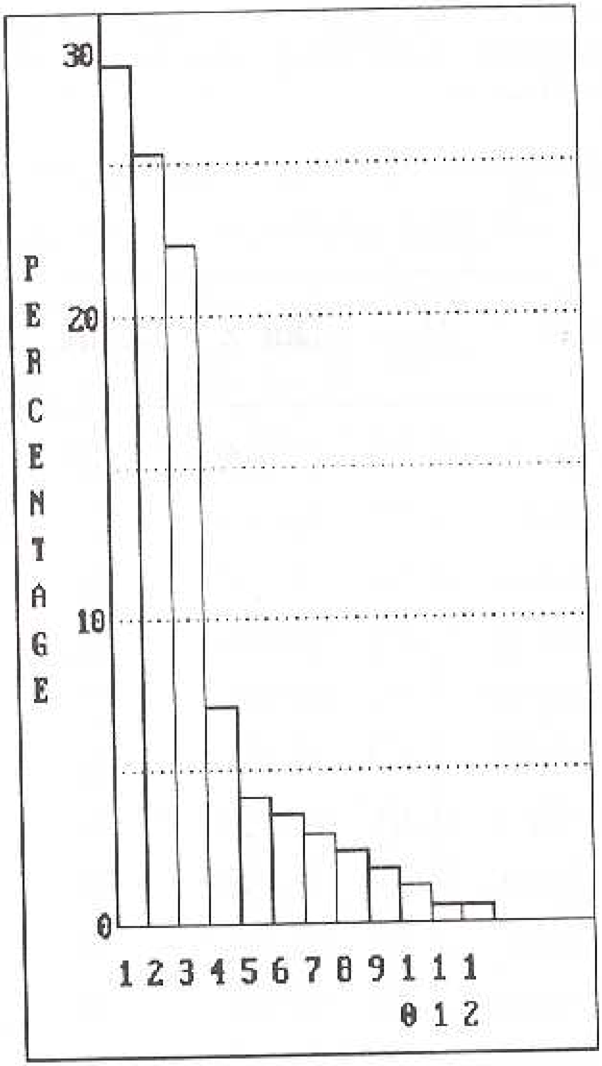
A Pareto chart shows various causes of the quality problem under consideration in order of frequency. The cumulative frequency curve can be shown on the Pareto chart, using the same scale or a different scale as the frequency on the vertical axis.

An example of hypothetical data concerning the causes of project delays is shown below:

Cause	%	Cum %	Count	Cum Ct
1 Scope ++	28.24	28.24	48	48
2 Unclear	25.29	53.53	43	91
3 Communic	22.35	75.88	38	129
4 Priority	7.06	82.94	12	141
5 Turnover	4.12	87.06	7	148
6 No staff	3.53	90.59	6	154
7 Conflict	2.94	93.53	5	159
8 Equipmnt	2.35	95.88	4	163
9 Material	1.76	97.65	3	166
10 Bad spcs	1.18	98.82	2	168
11 PM prmtd	0.59	99.41	1	169
12 Permit	0.59	100.00	1	170

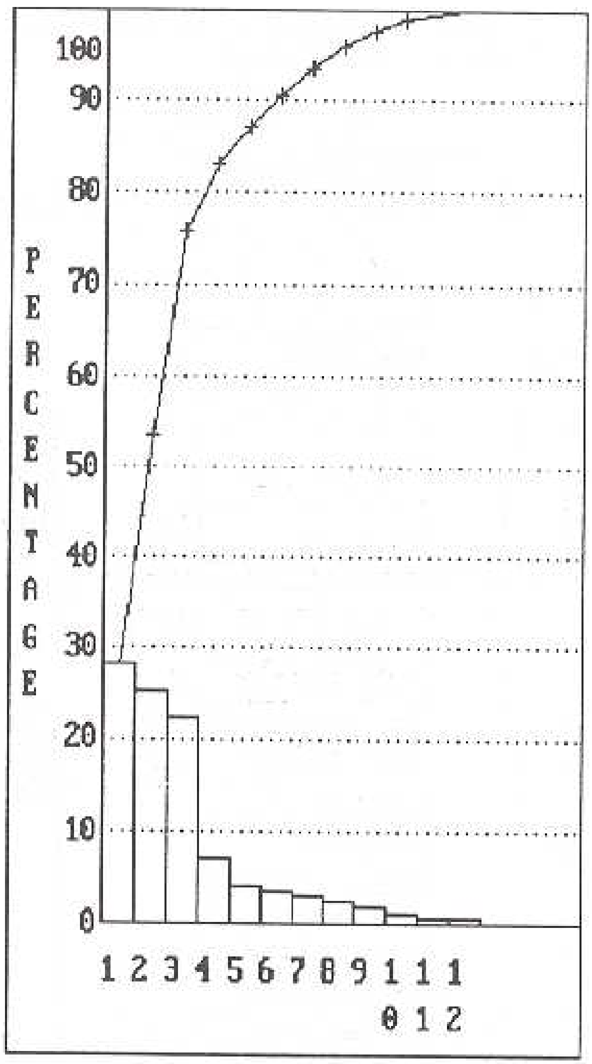
Pareto chart for causes of project delays

A Pareto chart depicting the data on the previous page for the causes of project delays is shown below:



Pareto chart for causes of project delays with cumulative frequency curve

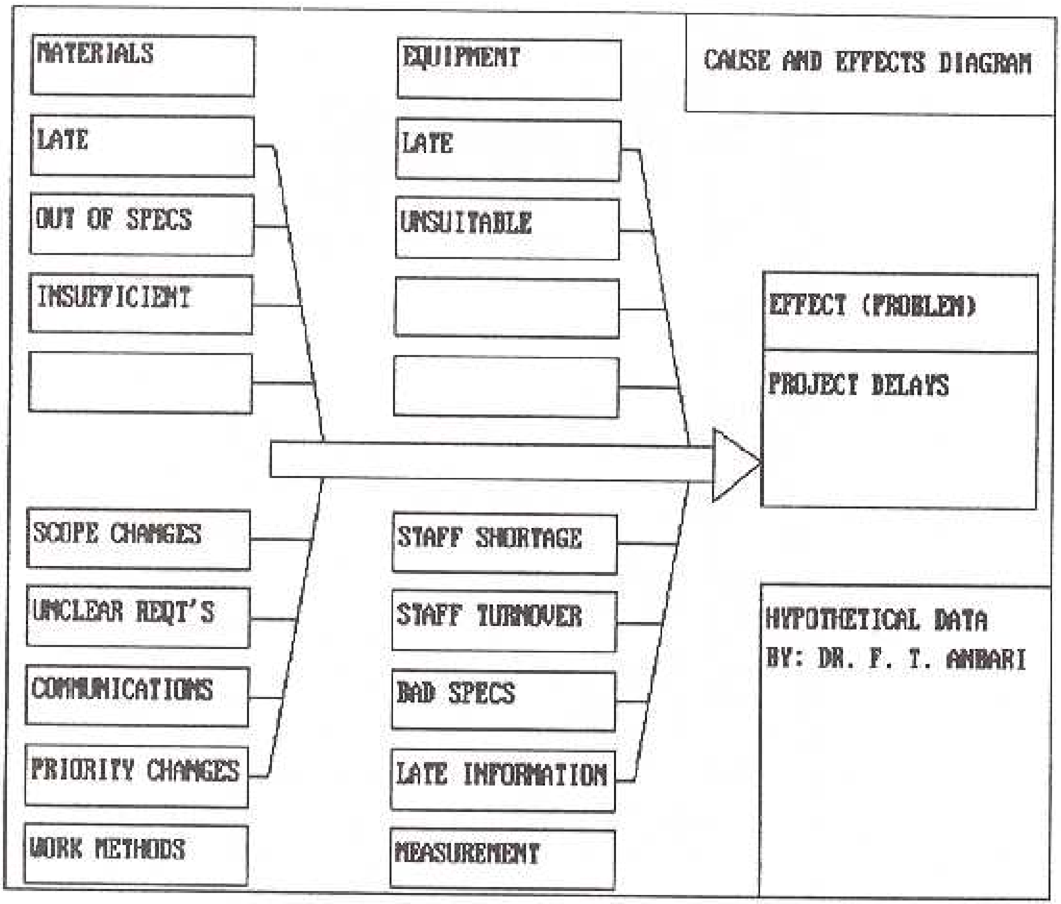
A Pareto chart depicting the causes of project delays is shown below, with the cumulative frequency curve, using the same scale as the frequency on the vertical axis.



Cause and effect diagram

The cause and effect diagram relates the effect, or the quality problem, to its possible causes. It is used to summarize the results of brain storming sessions and to focus the search for the root cause(s) of the quality problem. The cause and effect diagram allows various potential causes of the quality problem being studied to be grouped under selected major categories. It is also referred to as the fishbone diagram. It was developed by Dr. K. Ishikawa.

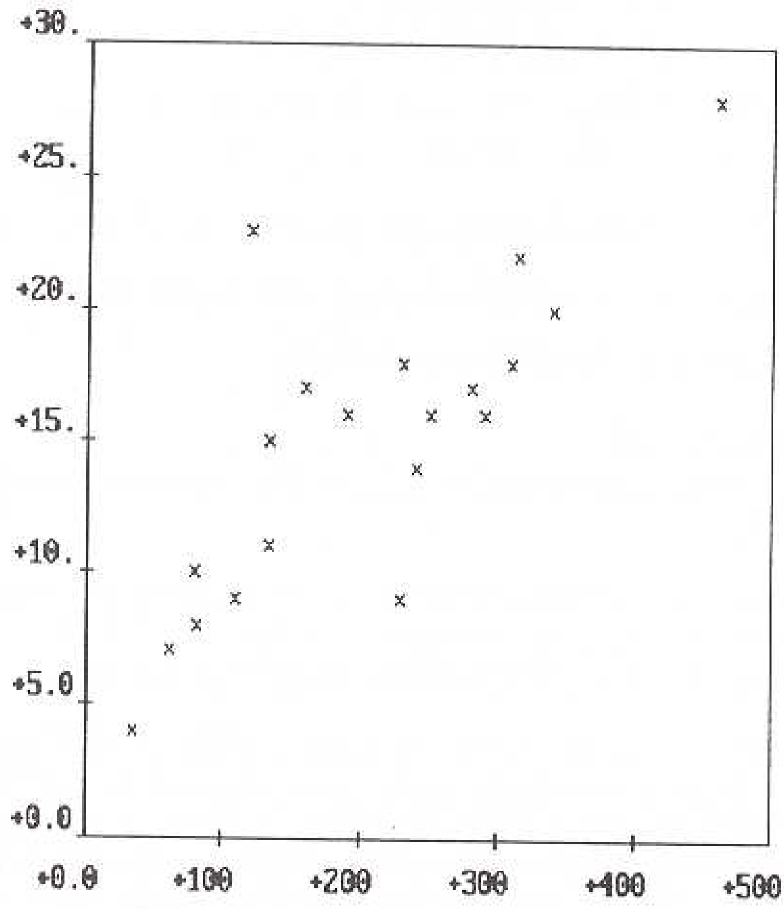
An example of a cause and effect diagram for hypothetical data concerning the causes of project delays is shown below:



Scatter diagram

The scatter diagram enhances the understanding of the type and strength of the relationship between two variables. The vertical axis, or Y-axis, shows the values of one variable, and the horizontal axis, or X-axis, shows the corresponding values of the other variable.

An example of a scatter diagram relating a particular company's annual profit, in millions of dollars, on the vertical axis to annual contract amount, in millions of dollars, on the horizontal axis is shown below:



Regression analysis

Regression analysis quantifies the type and shape of the relationship between two or more variables. The value of the dependent variable, Y, can be estimated based on the value(s) of the independent variable(s) X (or Xs). The simple linear regression model is:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

where $\hat{\beta}_0$ is the estimate of the intercept of the regression line with the Y-axis, and $\hat{\beta}_1$ is the estimate of the slope of the regression line.

The multiple linear regression model is:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \ldots$$

where $\hat{\beta}_0$ is the estimate of the intercept, and $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, ... are the estimates of the parameters of the regression model.

Application in cost management: The above model is used in parametric cost estimating, where $\hat{Y}$ is the estimate of the project cost and $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, ... are estimates of the cost parameters.

Correlation analysis

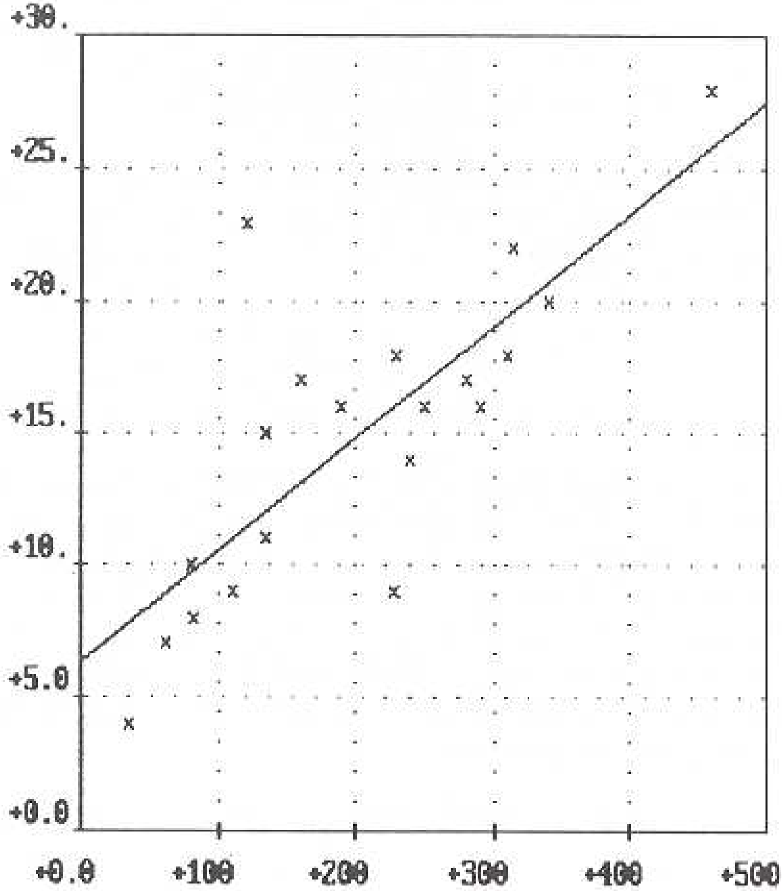
Correlation analysis quantifies the strength of the relationship between two or more variables.

The coefficient of determination, r^2, indicates the proportion of variability in Y that is explained by the linear regression model. An r^2 of "0" indicates no linear correlation, and an r^2 of "1" indicates perfect correlation.

The correlation coefficient, r, is the square root of the coefficient of determination. An r of "0" indicates no linear correlation, an r of "1" indicates perfect correlation. In simple linear regression, a negative r indicates inverse correlation, and a positive r indicates direct correlation. In that model, the sign of r should always be the same as the sign of the slope. In multiple linear regression, r is considered to have a positive sign always.

An example of a simple linear regression relating a particular company's annual profit, in millions of dollars, Y, to annual contract amount, in millions of dollars, X, is shown below. The simple linear regression model for this example is:

$$\hat{Y} = 6.32884 + 0.04232\,X$$



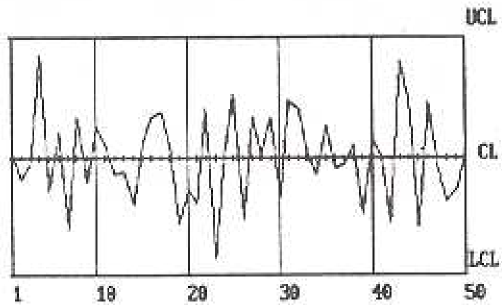
Control Chart

The control chart provides a view of the process characteristic of interest over time. Observations, or sample means are plotted on the chart. In standard control charts, the Centerline (CL) of the chart is the mean of the data, the Upper Control Limit (UCL) is set three standard deviations above the mean, and the Lower Control Limit (LCL) is set three standard deviations below the mean. These values are calculated from observations on the process itself. Therefore, the control chart represents the voice of the process. The control chart was developed by Dr. W. A. Shewhart. It allows differentiation between common cause variation and special cause variation.

Common Cause variation: This variation is caused by the total system, including planning, design, equipment selection, maintenance, personnel selection and training, etc. It can be referred to as system, random, or normal variation. Since management designs the system and has the authority to change it, management is generally considered to be responsible for system variation. Common cause variation is indicated when all plotted points fall within the control limits, with no trends, runs, cycles, or special patterns.

Special Cause variation: This variation is caused by causes outside the system, including human error, accidents, equipment break-down, etc. It can be referred to as assignable variation. Since this variation indicates a condition different than the way the system or process operates normally, it is generally considered to be the responsibility of the individual worker. Special cause variation is indicated when a plotted point, or points, fall outside the control limits, or when all plotted points fall within the control limits but have trends, runs, cycles, and/or special patterns.

A control chart in which only random variation is present, is shown below:



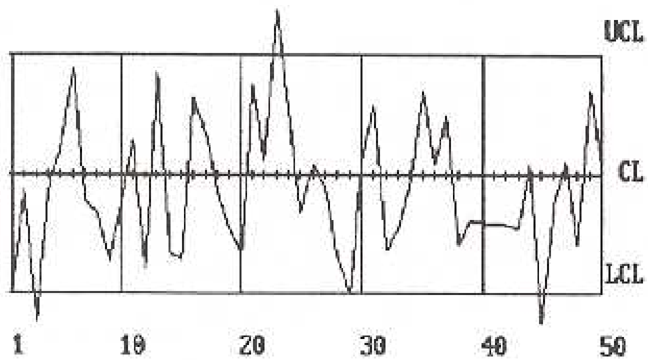
Control chart theory

The points plotted on a standard (Shewhart) control chart, are assumed to follow the normal distribution. If that assumption is not satisfied, the data can be mathematically normalized. Often it is possible to collect samples (usually called subgroups) of observations and plot their means on the control chart. Based on the central limit theorem, discussed earlier, the sample means tend toward a normal distribution, as the sample size increases. Samples of four or five observations are usually used.

Since the Upper Control Limit (UCL) is set three standard deviations above the mean, and the Lower Control Limit (LCL) is set three standard deviations below the mean, the distance between UCL and LCL is six standard deviations. When the process is in statistical control, the process is said to be stable, predictable, consistent, or in control. In this case, approximately 99.7% of the plotted points will be within the control limits. The remaining 0.3% (or 0.003, or 3 per thousand) of the plotted points will be outside the control limits: 0.15% above the UCL and 0.15% below the LCL. Points outside the control limits, while the process is indeed in control, are often referred to as false alarms.

Indications that the process is out of control

When the process is out of statistical control, the process is said to be unstable or unpredictable, or out of control. This is indicated when a plotted point, or points, fall outside the control limits. A control chart in which points fall outside the control limits is shown below, indicating that special cause variation is present:



The process is also said to be out of control when all plotted points fall within the control limits but have trends, runs, cycles, and/or special patterns. Based on probability theory, rules were developed to indicate trends and runs.

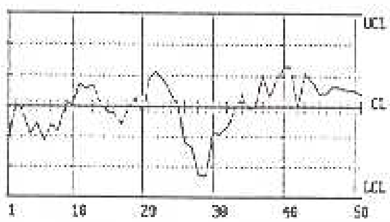
The following are widely used rules to indicate trends:

- Seven or eight consecutive points trending up on both sides of the centerline
- Seven or eight consecutive points trending down on both sides of the centerline

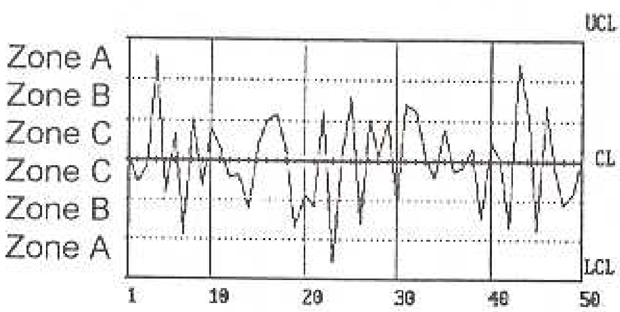
The following are widely used rules to indicate runs:

- Seven or eight consecutive points above the centerline
- Seven or eight consecutive points below the centerline

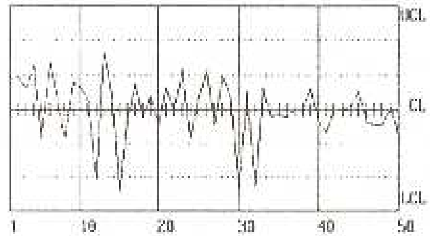
The above rules for trends and runs are sometimes referred to as the rule of seven. A control chart in which trends and runs are apparent is shown below, indicating that special cause variation is present:



Based on probability theory and the normal distribution, several rules were developed to indicate other special patterns. These rules break the control chart into zones as follows:



When fifteen consecutive points are found in zone C on both sides of the centerline, this is referred to as hugging the centerline. This situation indicates that the process is probably too good to be true. If significant process improvement has taken place, then the control limits should be re-calculated. Otherwise, problems may have occurred with measurement, data collection, or reporting. A control chart in which the points are hugging the centerline is shown below, indicating that special cause variation is present:



Based on probability theory, the normal distribution, and zone analysis, other rules were developed to indicate other special patterns. Recognizing these patterns can enhance the usefulness and interpretation of the control chart. However, these rules should be used with care, since an increase in the number of rules increases the probability of false alarms.

Standard (Shewhart) control charts with trend and run rules are considered by some to provide optimal indications of special cause variation while minimizing the probability of false alarms.

In recent years, other control charts have been developed and used successfully. These charts usually use the target value as the centerline. These charts include:

- Cumulative Sum (CuSum) chart, considered by some to be optimal
- Exponentially Weighted Moving Average (EWMA) chart
- Target (Rainbow) chart

These charts are used in Engineering Process Control (EPC) to regulate the process based on its output. In particular, the EWMA chart allows forecasting of the subsequent observation, and adjusting the process accordingly to keep the output close to the target value.

Process Capability

The control chart represents the voice of the process. It indicates whether the process is stable and predictable or not. However, it does not indicate whether the process is acceptable, adequate or capable of meeting specifications and customer requirements. Process capability studies combine the voice of the process with the voice of the customer, engineer, designer or manager.

Measures of process capability consider the following:

- Centerline (CL) is the mean of the process
- Upper Control Limit (UCL) is three standard deviations above the mean
- Lower Control Limit (LCL) is three standard deviations below the mean
- The width of the process is the distance between the UCL and the LCL. Therefore, it is always six standard deviations.
- Nominal Dimension (ND), or Target
- Upper Specifications Limit (USL), or Upper Tolerance Limit (UTL)
- Lower Specifications Limit (LSL) or Lower Tolerance Limit (LTL).

The following measures of process capability are widely used:

- The Capability Index:

$$C_p = \frac{\text{Specification Width}}{\text{Process Width}}$$

$$C_p = \frac{USL - LSL}{6\sigma}$$

- The Capability Index with Correction (k) for non-centrality:

$$C_{p_k} = \frac{|CL - \text{Closest Specification Limit}|}{3\sigma}$$

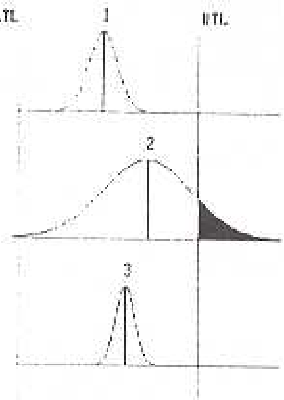
Examples

The following examples highlight differences among processes based on their variation and centering around the nominal dimension:

1) The process is somewhat centered with moderate amount of variation. Its measures of capability are approximately: $C_p = 2.2$ and $C_{p_k} = 2.0$

2) The process is not centered with high amount of variation. Its measures of capability are approximately: $C_p = 0.7$ and $C_{p_k} = 0.4$

3) The process is well centered with low amount of variation. Its measures of capability are approximately: $C_p = 3.1$ and $C_{p_k} = 2.6$



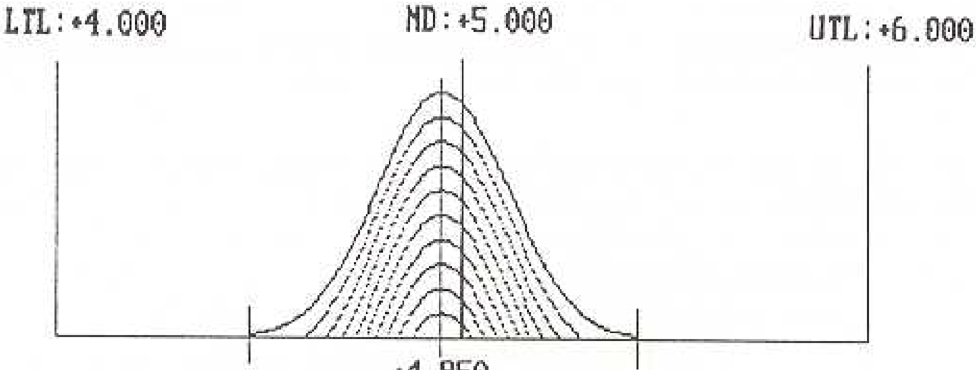
Maximum number of items out of specifications

From the information provided earlier on the normal curve, we can construct the following table for the maximum number of items out of specifications, or defective items, based on the number of standard deviations between the centerline to the closest specifications limit:

Closest Specs Limit to CL	C_{p_k}	Max. Proportion out of Specs	Max. Number of defectives per Billion
3 σ	1	0.002 7	2,700,000
4 σ	1.33	0.000 063	63,000
5 σ	1.67	0.000 000 57	570
6 σ	2	0.000 000 002	2

Application

The requirement of response time for a particular transaction is 5 seconds plus or minus 1 second. Data collected on this process indicates a mean of 4.95 seconds and a standard deviation of 0.16 seconds.



Process capability is calculated as follows:

$$C_P = \frac{USL - LSL}{6\sigma} = \frac{6 - 4}{6 \times 0.16} = 2.08$$

$$C_{Pk} = \frac{|CL - \text{Closest Specification Limit}|}{3\sigma} = \frac{|4.95 - 4|}{3 \times 0.16} = 1.98$$

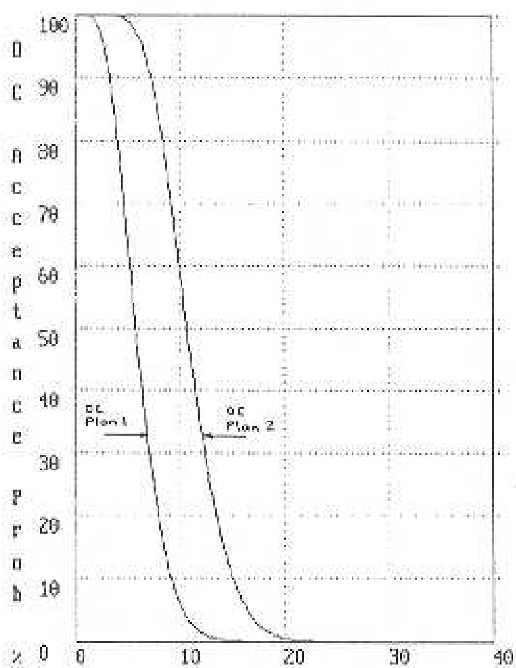
The C_P provides an upper limit for the C_{Pk} which is reached when the process is fully centered around the Nominal Dimension.

Acceptance Sampling

Acceptance sampling is an alternative to 100% inspection or no inspection. In this method, a **sample of size n** is inspected from the lot under consideration.

If the number of non-conforming, or defective, items in the sample is found to be equal to or less than the **acceptance number c**, the entire **lot** is accepted. If the number of non-conforming, or defective, items in the sample is found to be greater than the acceptance number c, the entire **lot** is rejected.

The Operating Characteristics Curve (OC Curve) for a particular **sampling plan** (with specific n and c) relates the probability of accepting a lot, shown on the vertical axis, to the percent defective in the lot, shown on the horizontal axis. These probabilities are based on the binomial probability distribution, discussed earlier. Two such plans are shown below: Plan 1: n = 100, c = 5, and Plan 2: n = 100, c = 10. Note that as the acceptance number c increases, the probability of accepting a lot with a certain percent defective increases.



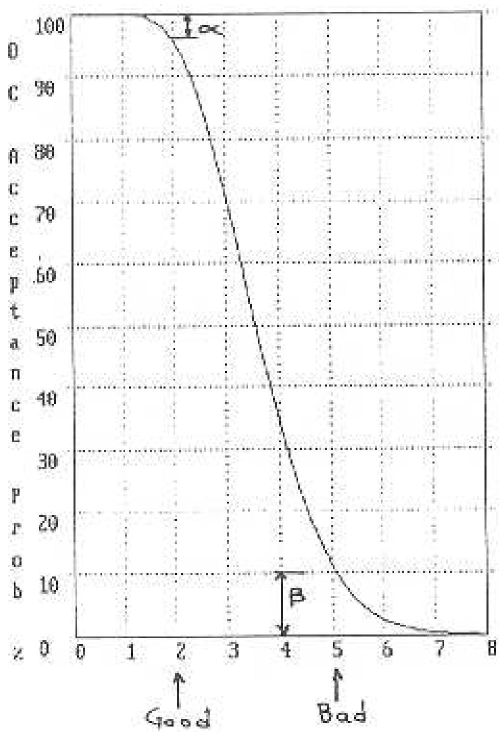
Acceptance sampling risks

In acceptance sampling, "good quality" is defined as a level of non-conformance that the consumer is willing to accept all the time. "Poor quality" is defined as a level of non-conformance that the consumer is willing to tolerate only a small percentage of the time.

Producer's Risk: The risk of rejecting a lot of good quality, based on a particular sampling plan. This is the probability of type I error, or α.

Consumer's Risk: The risk of accepting a lot of poor quality, based on a particular sampling plan. This is the probability of type II error, or β.

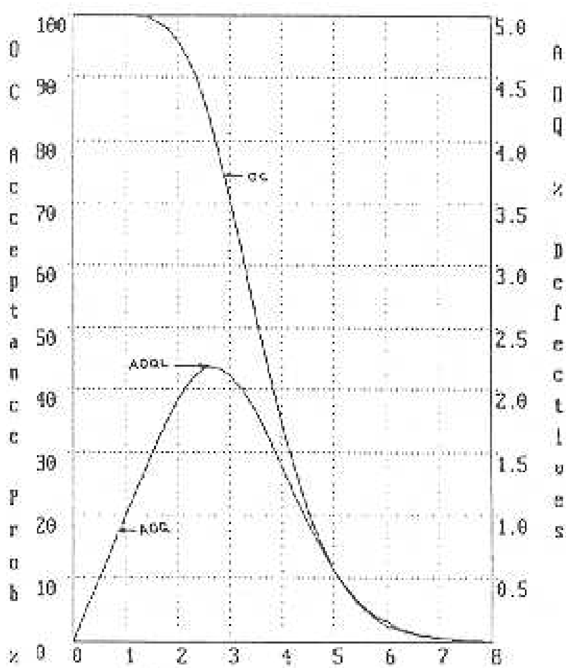
The following graph shows the OC curve for a sampling plan with n = 300 and c = 10. Assuming that for a particular application, 2% defective is considered to be good quality, whereas 5% defective is considered to be poor quality, the above risks are shown on the OC curve.



Average Outgoing Quality (AOQ): The average quality resulting (or outgoing) from the acceptance sampling process for many lots, based on a particular sampling plan. This includes all accepted lots and rejected lots. When a lot is rejected, the producer must perform 100% inspection on the entire lot, correct, or replace, all non-conforming items and give the "clean" lot back to the consumer. The rejected lots are assumed to have zero defects after inspection and correction.

Average Outgoing Quality Limit (AOQL): The maximum value of the Average Outgoing Quality (AOQ), based on a particular sampling plan. This represents the maximum level of "exposure" to the consumer of the sampling plan used in the acceptance sampling process.

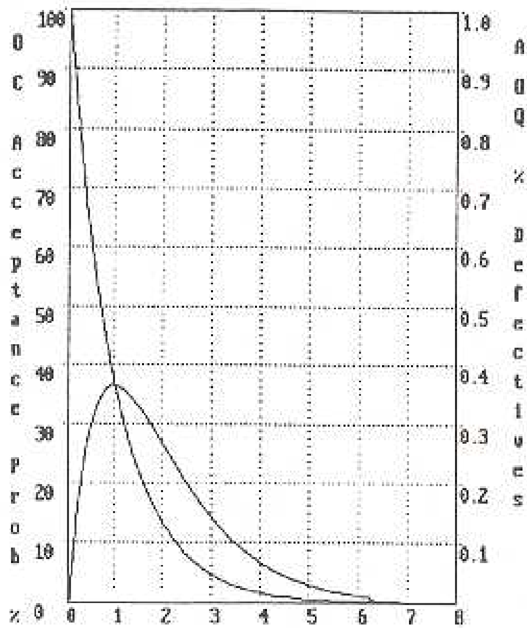
The following graph shows the OC curve and the AOQ curve for a sampling plan with n = 300 and c = 10. The **AOQL** is shown as the maximum value of the AOQ. Note that the values for AOQ are shown on the left vertical axis, whereas the values of AOQ and AOQL are shown on the right vertical axis. AOQL for this sampling plan is 2.18% resulting from the producer providing lots with 2.6% defective items.



Sampling plans can be based on single, double or multiple inspections. Probabilities associated with various sampling plans can be derived from the binomial probability distribution. An extensive number of sampling plans were formalized and tabulated by H. F. Dodge and H. G. Romig. Tables and graphs for sampling plans are published and are widely available. MIL-STD 105-E is published by the U.S. Government, and contains graphs for many sampling plans. Several Statistical Process Control (SPC) software packages can generate probabilities and graphs for a variety of sampling plans.

Acceptance sampling enjoyed wide popularity and usage for several decades. In recent years, it suffered numerous attacks as a guarantee or license for making defective items. Many organizations have shifted to using process capability measures, the C_p and the C_{p_k}, to combine the voice of the process with the voice of the customer, engineer, designer or manager.

The following graph shows the OC curve and the AOQ curve for a sampling plan with n = 100 and c = 0. It is an example of a "zero defects sampling plan" and may be more in line with current thinking. AOQL for this sampling plan is 0.37% resulting from the producer providing lots with 1.0% defective items.

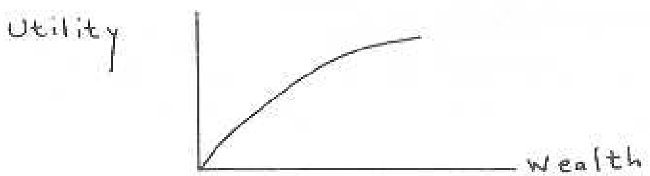


Utility Theory

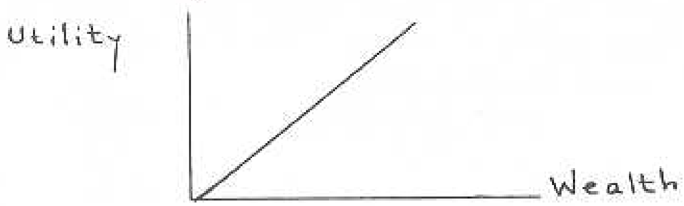
Utility theory provides a framework that helps in modeling a decision-maker's preference for various values of the variable under consideration. In risk management, the utility function depicts a decision-maker's propensity, disposition, or attitude, toward risk. Utility is measured on a scale from zero to one, and is shown on the vertical axis. On the horizontal axis, a measure of value is shown, such as money, wealth, timeliness, productivity or quality.

Three major attitudes toward risk are identified:

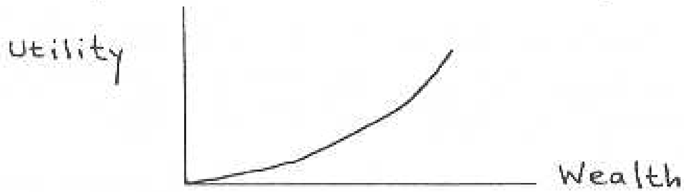
1) Risk Aversion: The decision maker is referred to as risk averse. The utility function increases at a decreasing rate.



2) Risk indifference: The decision maker is referred to as risk indifferent or risk neutral. The utility function increases at a constant rate.



3) Risk Seeking: The decision maker is referred to as risk seeker or risk lover. The utility function increases at an increasing rate.



As discussed earlier, the risk averse decision maker is likely to use the maximin, or the pessimist, criteria to select among alternatives. The risk neutral, or rational, decision maker is likely to use the expected value to select among alternatives. The risk seeker decision maker is likely to use the maximax, or the optimist, criteria to select among alternatives.

Among the many forms to represent the utility function are:
1) Exponential: $U(x) = -e^{-kx}$
2) Logarithmic: $U(x) = \log(x+b)$ where b is a constants.
3) Quadratic: $U(x) = a + bx - cx^2$ where a, b, and c are constants.

Utility theory was developed by J. Von Neumann and O. Morgenstern. It is more comprehensive than other approaches to decision analysis. However, its practical usefulness remains debatable.

Example

Given the following options:
A) Receive $100
B) Flip a fair coin for $200 or $0.

The risk averse is likely to take option A. The risk seeker is likely to take option B. The risk neutral is indifferent between the two options, since they have the same expected value, as shown below:

Outcome: x_i	Probability: $P(x_i)$	$x.P(x_i)$
$200	0.5	$100
$ 0	0.5	$ 0
	-----	---------
	1.0	$100

If the options were A) to receive $1,000,000 or B) to flip a fair coin for $2,000,000 or $0, many decision makers are likely to take option A. Thus, the amount at stake affects the decision maker's attitude toward risk.

Higher rewards make higher risks more acceptable. However, there is a threshold beyond which most decision makers avoid the risk. Decision makers are less likely to take risks they cannot afford.

Forecasting and Simulation

Forecasting plays an important role in project management. Forecasting methods include judgmental forecasting and quantitative techniques. Simulation can be used effectively in forecasting complex systems.

Judgmental forecasting include:

Expert group technique: A group of experts interact to reach a consensus forecast.

Delphi method: A questionnaire is distributed to a panel of experts. The responses are integrated and sent back to the same panel, along with a second questionnaire, developed based on the results of the first one. The results of the two questionnaires are used to generate a forecast and reach a decision about the issue(s) under consideration.

Quantitative forecasting techniques include:

Time series analysis: Historical data is used to develop a mathematical model to forecast the future. Forecasting procedures include the moving average and exponential smoothing. Linear trends, seasonal effects and cycles are widely used in forecasting models.

Regression analysis: The value of the dependent variable, Y, can be estimated based on the value(s) of the independent variable(s) X (or Xs), as discussed earlier.

Simulation

In simulation, a mathematical model is constructed to describe the real-life system. The model is run to obtain a view of the distribution of outcomes in the real world when various combinations of the factors in the system occur. Simulation can be used by itself to forecast future outcomes or with other methods such as the critical path method (CPM) or Linear Programming. Its objective is to gain better understanding of the various possible outcomes and their probabilities, not optimization. Simulation can be used in a variety of applications including project scheduling, cost estimating, operations analysis, system design, and training. It is most effective when it is too risky, time consuming or expensive to try various combinations in the real system.

Monte Carlo simulation

Concepts of simulation have been used for centuries in war games and experimentation. Monte Carlo simulation was developed by J. Von Neumann (developer of the architecture of the Central Processing Unit, the CPU), and achieved high levels of popularity and applications using the computer. This technique is implemented as follows:

1) Define the factors in the problem under consideration, and the relationship(s) among them. These factors may include activities completion times, equipment delivery time, labor rates, material cost, etc.

2) Determine the appropriate probability distributions underlying each factor in the problem. These distributions are usually based on historical data, experience and judgment. They can be developed empirically or based on the normal, binomial, Poisson, uniform, or other known probability distributions, discussed earlier. It is extremely important to use an appropriate probability distribution for each factor, since the results of the simulation are greately affected by the selection of these distributions.

3) Select intervals for random numbers for each factor in the problem. These intervals represent various outcomes of each factor and are based on the above probability distributions.

4) Generate random numbers using a computer random number generator, or a random numbers table. The following small set of random numbers was generated using Microsoft Excel:

0.618595	0.086213	0.557670	0.836101	0.278854
0.952061	0.402596	0.103519	0.867901	0.864697
0.397245	0.553257	0.751100	0.762059	0.395509
0.502290	0.201482	0.965725	0.597421	0.484371
0.214474	0.564187	0.088393	0.319485	0.868656

5) Conduct the first simulation by applying a new random number to each factor in the problem and converting that number into a value of an outcome for that factor based on the intervals for random numbers selected earlier.

6) Repeat the above step for the desired number of simulations. In some applications it is found that a 1,000 simulations are conducted, the results appear to stabilize.

7) Summarize the results of the simulation exercise, and prepare appropriate reports and graphs, to provide better understanding of the potential outcomes and their respective probabilities. This knowledge should enhance the organization's decision making abilities concerning future risks and opportunities.

Application in time management

Simulation of a project schedule can be performed in the manner described above. A probability distribution for the duration of each activity needs to be determined based on historical data, team experience, or experts' opinions. The simulation is then conducted using an appropriate computer software.

The results indicate probabilities of completion of the project within various given amounts of time, probabilities that various paths are critical, and the probability of each activity being on the critical path. A criticality index can be determined for each activity, based on the number of times it appeared on the critical path during the simulation process.

Schedule simulation takes into consideration the risk aspect of project scheduling. It avoids the criticism associated with the assumptions used in PERT, such as the beta distribution, the normal distribution, and independence of activities on the critical path. In particular, using simulation, all network paths are taken into account in determining probabilities of completion of the project within various given amounts of time. In PERT these probabilities are based on the critical path only, and sub-critical paths are not considered in probability calculations.

Depreciation

Each projects is an investment in which equipment may be purchased, facilities built and software developed. Depreciation allows the allocation of the cost of such assets to expense over the useful life of these assets. It can affect the tax position of the organization that made the investment.

Definitions

Cost: Initial cost of the investment
Salvage value: Estimated cash value of the asset at the end of its useful life
Depreciable cost: Initial cost of the asset less its salvage value
n: Expected useful life of the asset

There are four major methods for depreciation: Straight Line, Double Declining Balance, Sum of Years' Digits, and Units of Activity.

Straight Line

In this method, a uniform depreciation charge is used annually. The depreciation amount is calculated by dividing the depreciable cost of the asset over the useful life of the asset.

Double Declining Balance (DDB)

This is an accelerated depreciation method. In it, a uniform rate of depreciation is applied annually to the undepreciated value as of the end of the previous year. The rate used is double the depreciation rate used in the straight line method. In the first year, this depreciation rate is applied to the initial cost of the asset without subtracting the salvage value.

Sum of Years' Digits (SYD)

This is also an accelerated depreciation method, in which the depreciation charge is calculated as follows:

1. Calculate the sum of years' digits as: 1+2+3+4+...+n.

The same result can be obtained as follows:

$$SYD = n\left(\frac{n+1}{n}\right)$$

2. For each year, calculate the sum of years' digits fraction as the ratio of the number of remaining years over the sum of years' digits

3. Multiply the sum of years' digits fraction by depreciable cost.

Units of Activity (or Production)

In this method, the depreciation charge is calculated as follows:

1. Divide the depreciable cost over the expected useful life of the asset (in hours, miles, or similar units) to obtain the depreciation rate per unit

2. At the end of each year, multiply the depreciation rate per unit by the actual usage during that year.

In the units of activity (or production) method, the depreciation charge is not known in advance for each year. It is calculated after the conclusion of each year's activities.

An organization, or an individual, may chose the depreciation method that best fits their business needs. In depreciating real estate, only the straight line method is allowed.

An organization, or an individual, may switch the depreciation method, during the life of the asset, once and only to the straight line method.

Capital Budgeting

Projects are investments that compete for scarce capital resources of the organization. A strategic view should be taken to the selection of projects from competing proposed investments, including mandatory projects, in a manner that contributes most to the organization's objectives. There are many methods for ranking investment proposals. The Net Present Value, Internal Rate of Return and Payback Period are discussed below.

Net Present Value (NPV)

The Net Present Value (NPV) method is implemented as follows:

1) The expected future net cash flows (NCF) of the investment are discounted at the cost of capital (r) to the base year (present time) to obtain the Present Value (PV) of these flows.

2) The initial cost of the investment (I) is subtracted from the Present Value (PV) to obtain the Net Present Value (NPV) of the investment.

3) If the cost of the investment is spread over more than one year, future cost must also be discounted at the cost of capital to the base year.

4) Past expenditures are irrelevant to selection among alternative projects for the future. They are usually referred to as "sunk cost" to emphasize this point.

5) Calculation of the Net Present Value (NPV) is accomplished using the following formula:

$$NPV = \left[\frac{NCF_1}{(1+r)^1} + \frac{NCF_2}{(1+r)^2} + \dots + \frac{NCF_n}{(1+r)^n} \right] - I$$

where NCF_1, NCF_2, etc. are the net cash flows (NCF) for the respective years, r is the cost of capital, I is the initial cost of the investment, and n is the expected life of the project. The above formula can be expressed as follows:

$$NPV = \sum_{t=1}^{n} \frac{NCF_t}{(1+r)^t} - I$$

An organization should accept projects with a positive NPV and reject projects with a negative NPV.

In the NPV method, all future net cash flows are discounted at the cost of capital. Therefore, it is assumed that all future proceeds can be invested at the organization's the cost of capital.

Special care should be taken in analyzing infrastructure projects and mandatory projects, such as those required to meet a particular government regulation. Expected future net cash flows of these projects are often difficult to determine and the analysis results in negative NPV for these projects. However, they are usually required to be able to conduct the organization's business.

Internal Rate of Return (IRR)

The Internal Rate of Return (IRR) is the interest rate, or discount rate, that equates the present value of the expected future net cash flows to the initial cost of the investment. Alternately, IRR can be defined as the interest rate that causes the Net Present Value (NPV) of the investment to equal zero.

IRR is calculated using the following formula:

$$\left[\frac{NCF_1}{(1+r)^1} + \frac{NCF_2}{(1+r)^2} + \ldots + \frac{NCF_n}{(1+r)^n}\right] - I = 0$$

where all terms have the same definitions as those used in the NPV method.

The above formula can be expressed as follows:

$$\sum_{t=1}^{n} \frac{NCF_t}{(1+r)^t} - I = 0$$

IRR can be found using trial and error.

IRR can be used to rank projects. Projects with higher IRR should be accepted before projects with lower IRR. An organization should accept projects with an IRR that exceeds its cost of capital and reject projects with an IRR below its cost of capital, within the limitations of its borrowing and management capacities.

In the IRR method, it is assumed that all future proceeds can be invested at the IRR rate.

The comments mentioned earlier concerning infrastructure projects and mandatory projects, such as those required to meet a particular government regulation apply to the analysis conducted using the IRR method.

Payback Period

The payback period is the number of years needed to recover the initial cost of the investment from the expected future net cash flows resulting from the investment. Projects with shorter payback periods are considered more attractive than projects with longer payback periods. A cut-off number of years may be used to select or reject proposed investments based on this method.

The payback period method is widely used and easy to calculate. However, it can lead to the wrong decision, since it does not take into consideration the time value of money.

The expected future net cash flows of the investment can be discounted at the cost of capital to the base year (present time), then used in the payback period calculations. In this case, the results of project ranking based on the payback period method are consistent with the results obtained from NPV and IRR methods.

Some organizations use the payback period method in conjunction with the NPV and the IRR methods. In this case, NPV and IRR are used to indicate the profitability of the proposed project, whereas the payback period is used as an indicator of risk, since it shows the period of time during which the initial cost of the investment is at risk.

Comments on risk management

It is important to consider the risk associated with future cash flows. Risk is usually considered explicitly or implicitly in assessing future revenue increases, cost savings, operating cost, maintenance cost, etc.

Integration of cost management and risk management

Consider an investment of $900,000 to be made in the base year (the present time) in a particular project. The project has a 60% probability of performing well and generating a payoff of $2,000,000 in one year from now. The project has a 40% probability of performing poorly and generating an additional cost of $250,000 to retire the project in one year from now. Assuming a 10% discount rate, this investment is analyzed as follows:

Outcome: x_i	Probability: $P(x_i)$	$x.P(x_i)$
$2,000,000	0.6	$1,200,000
($ 250,000)	0.4	($ 100,000)
	-----	---------------
	1.0	$1,100,000

$$NPV = \left[\frac{NCF_1}{(1+r)^1}\right] - I = \left[\frac{1{,}100{,}000}{(1+0.1)^1}\right] - 900{,}000$$

NPV = 1,000,000 - 900,000 = $100,000

The NPV is positive and the investment should be made.

Break-Even Analysis

Break-even analysis is a profit planning method that considers the relationship between fixed cost, variable cost and revenues. The results of the analysis indicate the revenues required to cover the organization's total cost.

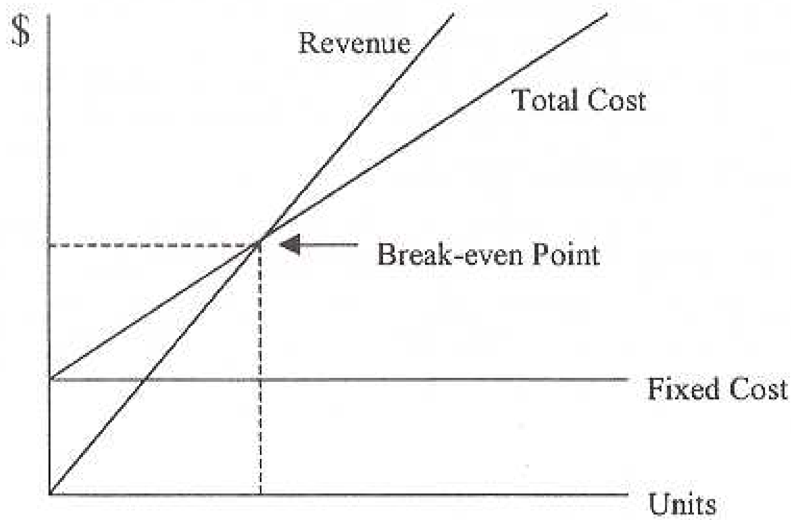
Fixed cost includes rental cost, depreciation, interest charges, salaries of the executive staff and similar costs.

Variable cost includes material cost, sales commissions, wages of production labor and similar costs.

Direct cost is the cost directly attributable to the project or production activities, such as labor and material. Therefore, it is mostly included in variable cost.

Indirect cost is the cost allocated, but not directly attributable, to the project or production activities, such as portions of rental cost, heating, air conditioning, and a portion of the salaries of the executive staff. This cost is sometimes referred to as general and administration (G&A) cost. Therefore, it is mostly included in fixed cost.

In the following break-even chart, the horizontal axis shows production units, and the vertical axis shows costs and revenues.



Earned Value Analysis

Overview

The Earned Value Analysis (EVA) method integrates three critical elements of project management: scope management, cost management and time management. It requires the periodic monitoring of actual expenditures and physical scope accomplishments. It supports the periodic evaluation of the time schedule and cost plan. It allows the project team/manager to adjust project strategy based on the cost and schedule requirements and performance of the project and the environment within which it is being conducted.

This powerful tool uses cost as the common measure of project cost and schedule performance. It allows the calculation of cost and schedule variances, performance indices, and forecasts of project cost at completion. It allows the measurement of cost in dollars, hours, worker-days, or any other similar quantity that can be used as a common measurement of the values associated with project work.

Definitions

The Earned Value Analysis method uses the following project parameters:

BCWS: Budgeted Cost of Work Scheduled. This is the budget baseline. It can be viewed as the value to be earned as a function of time.

BAC: Budget at Completion. This is the highest value of BCWS.

BCWP: Budgeted Cost of Work Performed. This is the earned value. It represents the amount of the budget earned by performing the work to a given point in time.

ACWP: Actual Cost of Work Performed. This is the Actual Cost spent to earn the value to a given point in time.

Cost performance is determined by comparing the actual cost of work performed to the budgeted cost of work performed (earned value). Schedule performance is determined by comparing the budgeted cost of work performed (earned value) to the budgeted cost of work scheduled (budget baseline). This is done by calculating the variances and the performance indices.

From the above definitions, the following is derived:

% Spent = ACWP/BAC
% Complete = BCWP/BAC

Variances

The following equations are used to calculate the variances:

CV = BCWP - ACWP: The Cost Variance is a measure of the conformance of actual cost of work performed to the budget

SV = BCWP - BCWS: The Schedule Variance is a measure of the conformance of actual progress to the schedule. SV can be translated to time units by dividing SV over the average BCWS per time period. The result is the SV in time units or the Time Variance (TV).

In these equations "0" indicates performance is on target. A negative value indicates poor performance. A positive value indicates good performance.

Performance indices

The following equations are used to calculate the performance indices:

CPI = BCWP / ACWP: The cost Performance Index is a measure of the conformance of actual cost of work performed to the budget

SPI = BCWP / BCWS: The Schedule Performance Index is a measure of the conformance of actual progress to the schedule

In these equations "1" indicates performance is on target. More than "1" indicates good performance. Less than "1" indicates poor performance.

Some organizations use the inverse of the equations given above to facilitate the use of the indices in forecasting.

Forecasting

The Earned Value Analysis method is useful in estimating the total cost of the project at completion and the cost to complete the remainder of the project, based on actual performance up to any given point in the project. The following equations are used to calculate these forecasts:

EAC = BAC / CPI:	Estimate at Completion
ETC = (BAC - BCWP) / CPI:	Estimate to Complete
VAC = BAC - EAC:	Variance at Completion

Forecasting of completion time

The Earned Value Analysis method can be used to calculate the project's total time estimate at completion (TEAC) and the time variance at completion (TVAC), based on the baseline schedule at completion (SAC) and actual performance up to any given point in the project. The following equations can be used to calculate these forecasts. These terms and equations have been used implicitly, but have not been documented as such in popular textbooks on the subject.

TEAC = SAC / SPI:	Time Estimate at Completion
TVAC = SAC - TEAC:	Time Variance at Completion

Applications

EVA can be applied to projects of various sizes. It can be applied at various levels of a project's Work Breakdown Structure (WBS) and to various cost components, such as labor, material, subcontractors, and other cost components.

Example

A project to implement a Wide Area Network (WAN) has a baseline budget of $200,000, and a baseline schedule of one year. As of the end of the fourth month of the project, 40% of the work has been completed at a cost of $75,000. Using the EVA method, the following can be calculated:

CV = $80,000 - $75,000 = $5,000
SV = $80,000 - $66,667 = $13,333
EAC = $200,000 / ($80,000 / $75,000) = $187,500
TEAC = 12 months / ($80,000 / $66,667) = 10 months

References

Cleland, D. I. and King, W. R. (eds.) *Project Management Handbook*, Second Edition. New York, NY: Van Nostrand Reinhold, 1988.

Deming, W. E. *Out of the Crisis*. Cambridge, MA: Massachusetts Institute of Technology, Center for Advanced Engineering Study, 1986.

Harrison, F. L. *Advanced Project Management*. England: Harts, 1983.

Ireland, L. *Quality Management*. Drexel Hill, PA: Project Management Institute, 1990.

Juran, J. M. *Juran on Quality by Design*. New York, NY: The Free Press, 1992.

Kerzner, H. *Project Management: A Systems Approach to Planning, Scheduling, and Controlling*, Fifth Edition. New York, NY: Van Nostrand Reinhold, 1995.

Project Management Institute (PMI). *A Guide to the Project Management Body of Knowledge (PMBOK)*. Upper Darby, PA: Project Management Institute, 1996.

Slemaker, C. M. *The Principles & Practice of Cost Schedule Control Systems*. Princeton, NJ: Petrocelli Books, 1985.

U.S. Department of Defense. *Instruction 5000.2, Defense Acquisition Management Policies and Procedures*. Washington, DC, 1991.

U.S. Department of Energy. *Cost/Schedule Control Systems Criteria for Contract Performance Measurement: Implementation Guide*. Washington, DC, 1980.

Wideman, M. *Risk Management*. Drexel Hill, PA: Project Management Institute, 1992.

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